

# Momentum and Mass Transfer by Eddy Diffusion in a Wetted-wall Channel

A. M. DHANAK

University of California, Berkeley, California

The role of eddy diffusion of mass (water vapor) and momentum was investigated in a specially devised wetted-wall channel in which the rippling of the liquid film was eliminated. The experimental measurements of the turbulent exchange coefficients for mass and momentum transport were carried out in a fully developed turbulent flow of air within the range of Reynolds numbers of 8,000 to 160,000. A correlation with Reynolds number revealed an approximately linear relationship of the eddy diffusivities to Reynolds number [Equation (4)]. From the hot-wire measurements it was found that within the main portion of the turbulent core eddy diffusivities remained fairly constant.

In many industrial processes such as humidification, drying, mixing, etc., turbulence exerts a marked influence on the transport of material or heat. In such processes the state of motion of the fluid, i.e., whether laminar or turbulent, is of primary importance. Furthermore, for design engineers estimation of the basic sizes of equipment in these processes requires a knowledge of the resistance to mass or heat transfer. Approximately 30 to 60% of the total resistance is generally contributed by the eddy diffusion in the turbulent core of the main flow.

Although the general subject of turbulence has been under intensive investigation for over half a century, little or nothing is known concerning the mechanism governing turbulent mass transfer. A survey of literature reveals that experimental data on turbulent mass transfer, particularly in pressure flow such as in a channel or a duct, are scarce. A number of studies have been carried out on mass transfer in free turbulence, as in the case of a jet or a wake, in which the transport phenomena governing turbulent diffusion of heat, mass, or momentum are perhaps analogous in nature. However, in the case of pressure flows this may not be so, especially at points outside a portion of the central turbulent core. Isakoff and Drew (1) obtained correlations for eddy diffusivities of heat and momentum from their measurements on heated mercury flowing turbulently in a tube. The ratio

of these diffusivities was shown to vary across the cross section and with Reynolds number. The results of Schwarz and Hoelscher (2) indicate that in the main portion of the turbulent core the turbulent Schmidt number is fairly constant and that its order of magnitude is between 1 and 2 in the core except near the center of the pipe. Sherwood and Woertz (3) have reported data correlating the overall eddy diffusivity of mass transfer in turbulent flow between parallel plates. The value of Schmidt number obtained by them is close to 0.7. Mickelsen (4) found from the hot wire and the helium-diffusion data in a round duct that the proportionality factor between the Lagrangian and Eulerian correlation coefficients was approximately 0.6. Similar results have been reported by Towle and Sherwood (5), Schlenger and Sage (6), and Kalinske and Pien (7).

The present study (8), with a particular emphasis on the elimination of rippling effects, is devised as an attempt (1) to substantiate Sherwood and Woertz's conclusions, (2) to formulate expressions for mass and momentum eddy diffusivities based on theoretical considerations, and (3) to present an experimental correlation for mass transfer eddy diffusivity.

## APPARATUS

The schematic arrangement and the overall view of experimental equipment are represented in Figures 1 and 3. A horizontal rectangular duct with an aspect ratio of 6:1 served as a mass transfer channel, details of which are shown in Figure 2. It

comprised the wetted side walls, a 1-in.-thick Plexiglass cover on the top, and a 1-in.-thick plywood bottom. The Plexiglass cover was provided with twenty-one Pitot-tube traverse tracks, and the plywood base had  $\frac{1}{4}$ - by  $\frac{1}{4}$ -in. troughs to carry drain water from the wetted plates. Narrow strips of aluminum sheet metal were placed at an angle to the bottom in order to prevent water from wetting the channel floor. The mass transfer plates were specially designed so as to eliminate the rippling effect of the liquid film. The existence of such a rippling effect has been reported frequently in the literature and is a cause for concern for the obvious reason that it tends to introduce a source of turbulence at the wall. Various methods were tried in an attempt to devise a wetted wall which would constitute a basically good mass transfer surface. The involvement of a felt-covered plate which gave smooth liquid film without the associated rippling effect came about after much experimentation with a number of materials. Green billiard felt 1/16-in. thick was glued to the 1/16-in. brass-sheet-metal plate with Goodyear Plyobond. The whole plate with the felt was kept under a heavy press for 48 hrs. in order for the bonding substance to set tightly. A brass tube soldered on the back of the plate fed liquid to the felt surface through a narrow slit in the brass. Adjustable clamps with set screws were placed at intervals along the feed tube to give a uniform flow of liquid by partially closing or opening the slit through clamping action. Uniformity of saturation of felt was judged by shining a floodlight at an angle through the Plexiglass cover. A total of six plates each 3 ft. long were used, comprising a 6-ft.-long mass transfer calming section and a 3-ft.-long test section. The mass transfer channel was carefully aligned and

A. M. Dhanak is at present with General Electric Company, Schenectady, New York.

leveled with a carpenter's level when assembled. The inlet to the channel was preceded in order by a 3-ft.-long dry conditioning section, an 18-mesh turbulence screen, an egg-crate section, a converging section, and a transition section. The exhaust portion of the channel comprised a 1-ft.-long exit duct and a 7-deg. diffuser section. All parts of the flow system except the channel were constructed of 24-gauge galvanized-iron sheet metal. A screen holder was provided for introducing the screen into place, and the flow of air was supplied by a centrifugal blower driven by a 5-hp. d.c. motor. Variable motor speed was achieved by separate field and armature control. The suction side of the blower consisted of a plenum chamber and a filter box, the box containing three layers of commercial glass-wool panels each 1-in. thick. Air from the blower was discharged into a 9-in.-round duct through a 9-in. control valve. A 4½-in. orifice was installed in the duct following an egg-crate section placed about 8 pipe diameters apart.

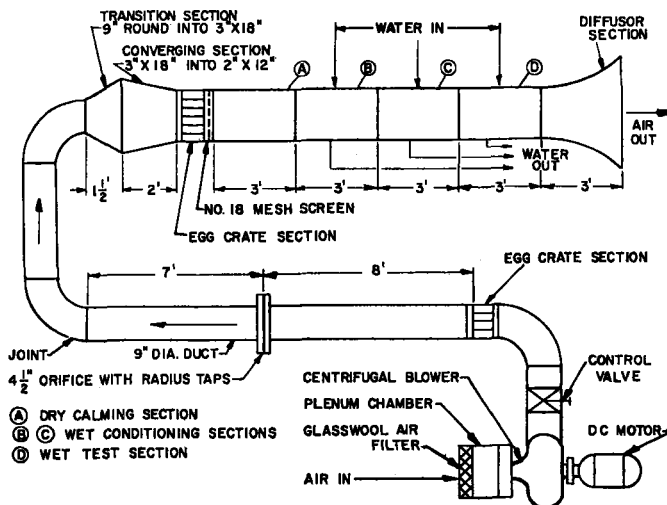
Constant-level head tanks supplied water to the six felt plates through a feeder pipe and a set of two-way valves. The level of the water in the head tanks was maintained constant by means of an overflow pipe inside the tanks. The drain water from the felt plates was carried back to sump tanks. The function of the two-way valves was to switch from one tank to the other for metering purposes. The circuit was completed by laboratory circulating pumps which served to supply water to the head tanks. Before returning to the metering sump tanks, water was passed through a 12-in.-diam. stilling well in order to minimize pulsations of flow. Air from the channel was prevented from leaking through the drain-water passage by means of a 6-in.-high water-seal drain pipe. Water from the plate entered at the bottom of the pipe and overflowed from the top into the line carrying it to the sump tank. All joints in the water lines were sealed by a coating of wax or Glyptal paint, and Garlock gaskets (1/16 in.) were used between flat surfaces.

Water was heated to a constant temperature by means of a 500-watt immersion type of heater placed in each of the two head tanks. The temperature was controlled and maintained automatically by means of a Fenwall thermoswitch.

## TEST PROCEDURE

## General

Before each day's runs were started, a visual inspection of the felt plates was made through the Plexiglass top to determine whether the plates were thoroughly wetted. Frequently it was found necessary to bleed all water lines and to rewet the plates, as air in the water tended to coagulate, forming air bubbles which restricted the flow of water to the plates. The plates were wetted by gently rubbing a clean piece of wet rag over the felt surface. When the experiment was not in progress, and during the night, plates were supplied with water from the city water line. Before a run the tanks were filled with fresh water from the city line, as it had been confirmed by taking one run with distilled water and repeating it with fresh city water that distilled water was not necessary, the



**Fig 1. Schematic diagram of experimental equipment.**

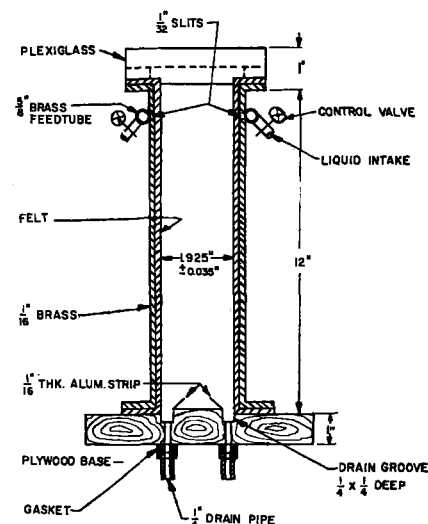
results in terms of concentrations traverse and evaporation rates obtained in both cases having been found to be substantially the same. Finer adjustment of air flow rate was accomplished with the 9-in. control valve. The sampling lines were allowed to be flushed out for a period of 20 to 30 min.

Before actual test measurements were started, uniformity of saturation was determined, and any water that might be swept away from the trough by air at relatively high velocities was detected. Inspection was also made for leaks in the water lines.

## Measurements

The over-all test program was divided into two parts: part *A*, asymmetric case (one wall wetted), and part *B*, symmetric case (both walls wetted). For each case twelve runs were taken with mean velocity in the channel varied from 6 to 120 ft./sec. The corresponding range of Reynolds numbers was from approximately 8,000 to 160,000. The calculation of Reynolds number was based on twice the mean channel width, with the width varying somewhat all along the length of the channel, as determined by the contact indicator (used with the Pitot tube) at several points along the channel. A variation of about  $\pm 0.035$  in. with a mean value at 1.925 in. was noted.

Friction factors were determined experimentally with a micromanometer from the pressure-drop measurements for both dry and wet runs. Velocity profiles were obtained by making Pitot-tube traverses midway in the test section along a line connecting the centers of the felt plates. The traverses were taken at nine positions spaced  $\frac{1}{4}$  in. apart except in the region near the walls, where  $\frac{1}{8}$ -in. separation was utilized. The velocity heads were measured by means of the micromanometer. Fifteen copper-constantan thermocouples were located on the inlet and outlet water lines as well as inside the channel. The temperature readings, recorded at the beginning and end of each run, were obtained with a Brown electronic potentiometer, the tem-



**Fig. 2. Channel cross-sectional details.**

perature of the inlet water to the plate being fixed at  $78^{\circ} \pm 2^{\circ}\text{F}$ . for all runs. Local temperature of the liquid film was measured by means of a traversing thermocouple device which consisted of a small probe with a thermocouple head. Over-all mass balance on the system was made for each run by measuring inlet and outlet air humidities and the evaporation rates from the walls. The mass-balance discrepancies were found to be within  $\pm 15\%$ .

The concentration profiles were obtained by drawing the sample through the impact hole of the Pitot tube with an aspirator and passing it through the two 150-mm. U tubes in series filled with anhydrous calcium sulfate (Drierite). A wet-test meter with a least count of 0.01 cu. ft. was used for metering the sampling rate, which was adjusted to correspond approximately to the speed of air at the point of sampling. This rate ranged from 0.005 to 0.05 cu. ft./min. The upper limit was fixed by the pressure drop in the sampling line. The gas sample was allowed to run through the U tubes for a period varying from 10 to

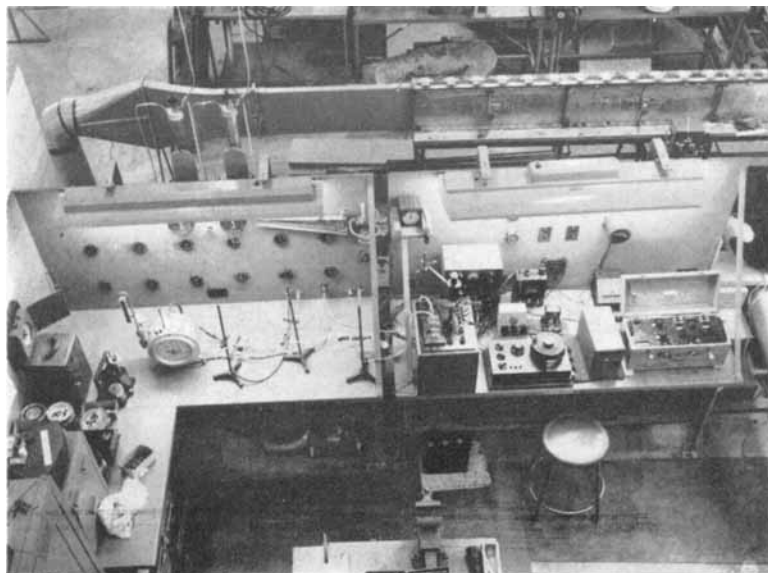


Fig. 3. Over-all view of the experimental setup.

20 min., and changes in the weights of the U tubes were noted. Generally, one tube was all that was necessary to absorb completely the water content in the gas. The sampling rate was carefully adjusted to the optimum value for complete absorption of water vapor by by-passing part of the sample. A precision balance with its arm calibrated to 0.1 mg. was used for weighing the U tubes. A small dish filled with anhydrous calcium sulfate (Drierite) and placed inside the balance ensured the presence of dry air within the enclosure, for the U tube might otherwise pick up moisture from the air. The weights were not calibrated, as the same pieces of weights were used for "before and after" weighings. Sample traverses were made to determine concentration distribution from top to bottom of the channel. Determinations were also made of axial concentration distributions at three velocities (Reynolds numbers of 45,000, 99,000, and 161,000), as shown in Figure 5.

The evaporation rate was determined for each plate separately by noting the time required for the depletion of 133 cc. of water in the metering tank. This was done automatically by means of a special triggering device, together with an electric timer which had an accuracy of 0.1 sec. Time readings for these rates were repeated until they remained fairly constant, three such readings being recorded and the average taken. At extremely high velocity (above 100 ft./sec.) of air in the channel, some water was blown away by the air. This was collected at the discharge outlet and weighed, and a correction was made in the evaporation-rate measurement.

By means of commercial hot-wire (constant current) and anemometer instruments, turbulent intensities,  $\sqrt{u'^2}$  and  $\sqrt{v'^2}$  and their distributions across the test section were determined. The hot-wire probe was carefully aligned with respect to the flow direction. Maximum noise level for all measurements amounted to less than 4% of the hot-wire output voltage and remained fairly constant throughout the experiment.

Velocity traverses were made at the same

positions as the hot wire. Figure 4 shows an enlarged picture of a specially constructed streamlined Pitot tube, which was used for velocity and concentration mappings. It was mounted on a horizontal traversing platform which moved transversely to the plates. The traversing mechanism consisted of a vernier depth gauge which could be read accurately to 0.0001 in. The vertical movement of the Pitot tube was achieved by means of a vernier gauge mounted on the movable platform, the reading of the gauge being accurate to 0.001 ft. The Pitot tube was positioned in reference to one of the wetted walls, and the zero reference reading was determined by contacting the Pitot tube with the wetted wall. An indication of exact contact with the water film was given by an electric light actuated by a miniature relay. Readings of static and impact pressures of the Pitot tube were obtained by means of a water micromanometer, which had a vernier movement of a rhodium-platinum point contacting the water surface in a small gold-plated cup. Contact of the manometer probe with the water surface was signaled electronically in a manner similar to the Pitot-tube zero indicator described above. The vernier movement had an accuracy of  $\pm 0.0001$  in.

Air flow rate was determined from the pressure drop across a  $4\frac{1}{4}$ -in. sharp-edged orifice installed in the 9-in. line. At high flow rates an Ellison-type inclined draft gauge was used. For low velocity runs (below 1-in. pressure drop) the micromanometer was employed. The orifice was precalibrated in place against the integrated traverse readings of the Pitot tube, and the Ellison manometers were calibrated against the micromanometer. Normal corrections were made in the manometer readings to account for the variation of specific gravity of the Ellison fluid with temperature.

## RESULTS

### Momentum Transfer

Estimation of eddy viscosity and eddy diffusivity requires knowledge of the velocity gradient and friction factor. The former can be obtained from the velocity traverse, and the latter from the pressure-drop measurements or Blasius's curve for smooth walls. Measurement of a gradient from the velocity distribution, however, would involve large uncertainties; therefore, the equation of shear stress is integrated. The result is expressed in terms of velocity deficiency, and the integration is carried out on the assumption that the eddy viscosity  $\epsilon$  is constant in the channel cross section.\* Figure 6 shows a logarithmic plot of  $(U_m - U)$  vs.  $(b - y)$ . From this and Figure 7, for friction factors, eddy viscosity is determined, and the variation of eddy viscosity with Reynolds number is depicted in Figure 8. The friction factor for wetted walls appears to lie approximately near Blasius's curve for smooth wall. In the

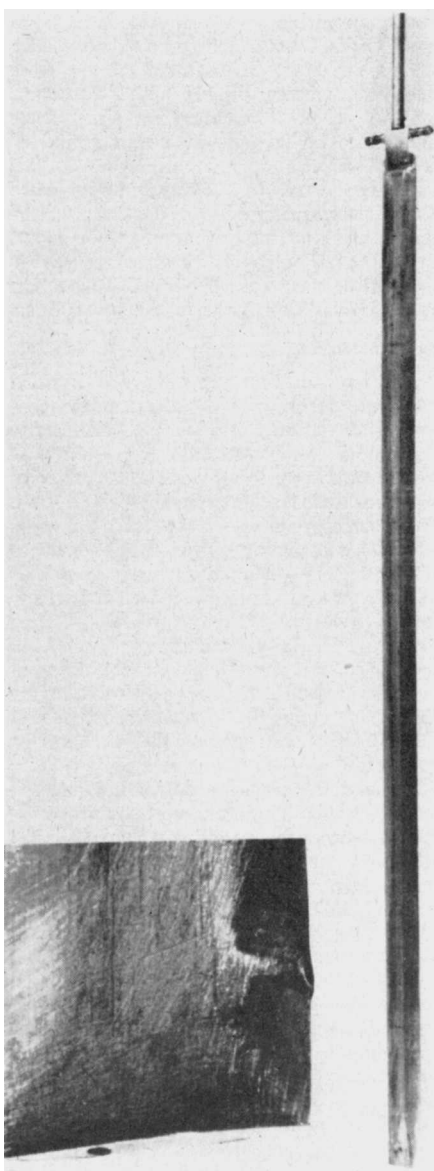


Fig. 4. Photograph of streamlined pitot tube.

\*This assumption appears to be fairly valid for the central portion of the turbulent core. A variation of approximately 15% in the value of the eddy viscosity is evidenced in this region from the data of Isakoff and Drew (1).

dry case, where the walls were not wetted, the friction factors are, in general, high—by as much as 50%. The eddy viscosities in Figure 8 are roughly 50% lower in comparison with the ones obtained by Woertz in a similar fashion. The linear variation of eddy viscosity with Reynolds number is, however, apparent in both cases, but the reason for the discrepancy is not obvious at first. It might be suspected that wall conditions played some role, and, although no mention is made of this fact by Woertz, it is generally reported in the literature (2, 9) that some amount of rippling always occurs when gas flows past a liquid film. This effect would change markedly the wall turbulence and, in general, induce more eddies for transport. This is borne out somewhat by comparing Woertz's data on percentage of turbulence with the present case where rippling has been eliminated. Woertz's data represent, in general, a higher percentage of turbulence increasing sharply toward the wall. To consider the relative effects of dry and wet walls on turbulence, hot-wire data were taken with one wall wet and the other dry. In Figure 9 percentage of turbulence in the wet half of the cross section appears, in general, to be lower than in the dry side. Both the turbulence and the velocity curves have shifted more toward the wet wall, and further substantiation is necessary. The effect of wall condition on the velocity distribution is evidenced in Figure 10, where the shift toward the wet wall is more noticeable.

From the hot-wire measurements and the friction factor, a knowledge of the magnitude of the correlation coefficient  $R$  between root-mean-square values of  $u'$  and  $v'$  can be gained. The results of these measurements are presented in Figure 11, where, for comparison, experimental data of Reichardt (10), Watten-dorf (11), and Woertz (3) are also shown plotted. The comparison data are for lower Reynolds numbers. Although the correlation coefficient is higher, the general shape of the curve agrees quite well with these other results. The calculation of correlation coefficient is based on the assumption that the turbulence is isotropic in the main core of the channel. The experimentally measured magnitudes of the turbulent intensities in  $x$  and  $y$  directions ( $x$  being parallel to the main flow and  $y$  perpendicular to the walls) point to such a conclusion (Figure 12), and Watten-dorf's data obtained for air flow in a similar channel also support this observation.

Validity of the momentum-transfer data is supported by the agreement with the universal velocity distribution of Figure 13 ( $u^+$  vs.  $y^+$ ), the comparison being shown for only the turbulent regime ( $y^+ > 30$ ). The agreement is also satisfactory with the data of Nikuradse (12), Deissler (13), Laufer (14), and Corcoran and Sage (15).

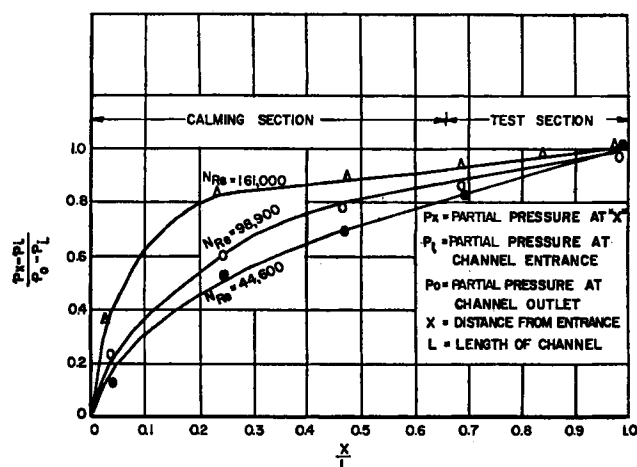


Fig. 5. Axial concentration distribution.

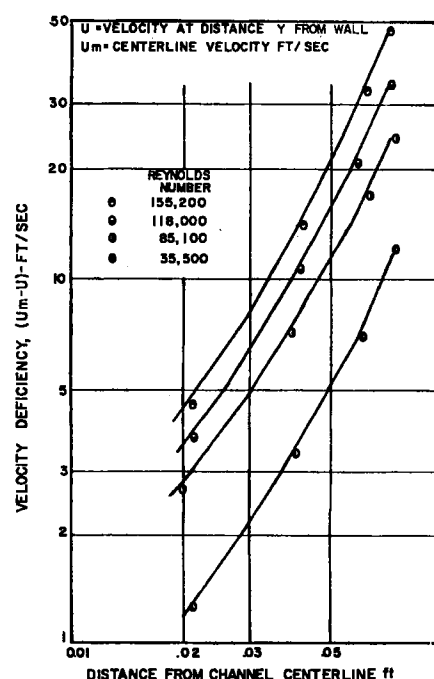


Fig. 6. Typical velocity deficiency curves.

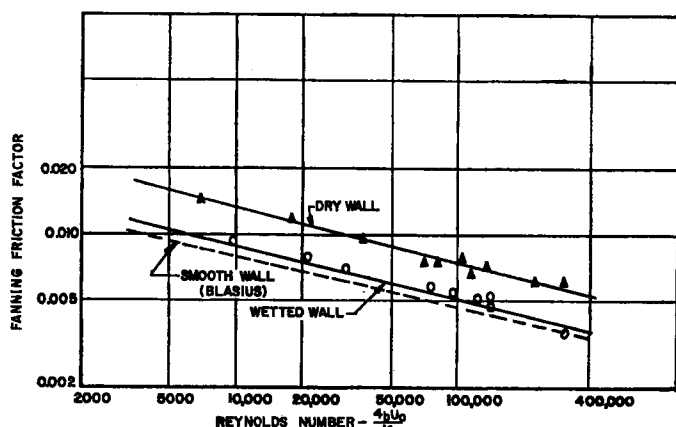


Fig. 7. Wall friction factors from pressure drop data.

**Asymmetric Case (Series M).** This part of the experiment was conducted with one wall wetted and the other dry. Concentration readings were obtained in terms of mole fraction of water vapor in air and converted into partial pressures (Figure 14). The wall concentration corresponds to the vapor pressure of the saturated air at the center-point temperature of the wetted wall, which was estimated from the knowledge of inlet and outlet water temperatures ( $t_1$  and  $t_2$ ) by the following formula:

$$t_c = t_2 + \frac{t_1 - t_2}{10}$$

The equation was determined by taking temperature traverses on the wall along the height of the channel for several air velocities by means of a traversing thermocouple probe. This formula which generally predicted the midway temperature within approximately 1°F., was confirmed from several spot checks of the temperature during the experiment.

The equation used for the diffusivity determination is given by

$$D + E = -\frac{N_A R_G T}{(dp_w/dy)}$$

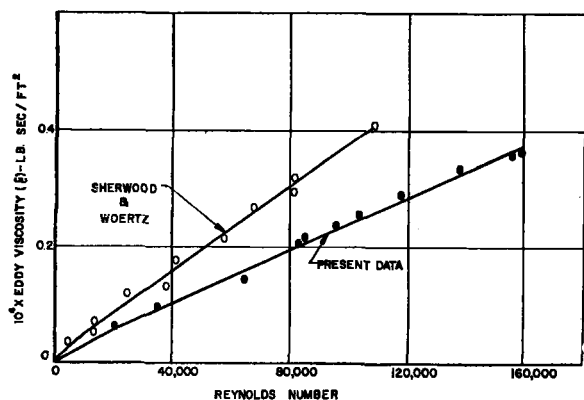


Fig. 8. Variation of eddy viscosity with Reynolds number.

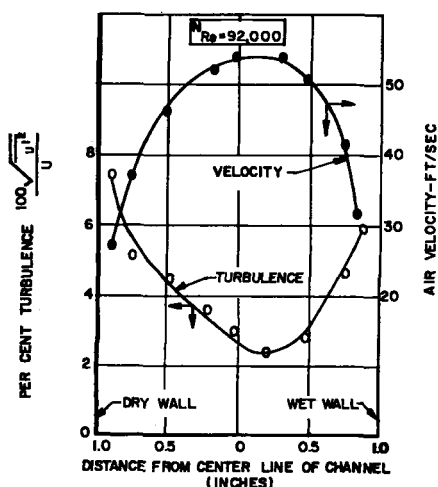


Fig. 9. Typical distribution of percent turbulence.

The molecular diffusion coefficient  $D$  was computed from the equation of Hirschfelder (16) and was usually much smaller (at the most, about 5%) in comparison with the eddy diffusivity  $E$ . The total diffusivity (eddy + molecular) was determined from the slope of the concentration profile taken arbitrarily at a point in the turbulent core about two thirds from the center. The underlying assumptions for this method of determination are (1) mass transfer by eddy diffusion in axial and vertical directions is negligible, (2) the diffusivity is constant in the turbulent core, and (3) mass transfer by axial convection of water vapor by the main flow in the region between the wall and the edge of the turbulent core is small in comparison with the total mass transfer rate from the wall. The first assumption is only partly correct, as the experimental data showed that slight gradients did indeed exist in the direction of flow as well as in the vertical direction. Neglecting eddy diffusion in these directions, however, had no serious effect on the computation of eddy diffusivity, since the gradients in the  $x$  and  $z$  directions were appreciably smaller in comparison with the one in the  $y$  direction. The slope of the concentration profile, for

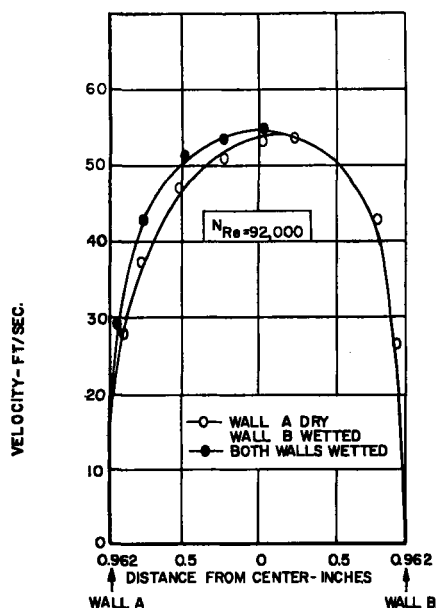


Fig. 10. Effect of wall condition on velocity distribution.

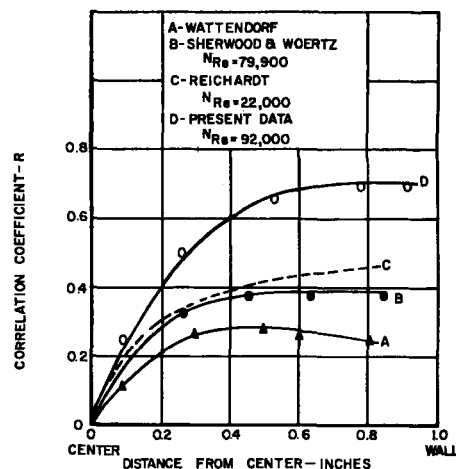


Fig. 11. Typical correlation of deviating velocities  $u'$  and  $v'$ .

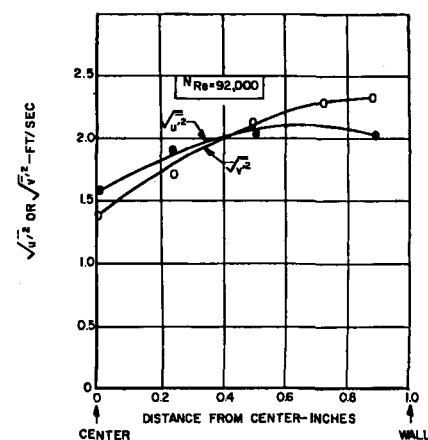


Fig. 12. Comparison of  $\sqrt{u'^2}$  and  $\sqrt{v'^2}$  from typical hot-wire data.

the purpose of computing the eddy diffusivity, was taken arbitrarily at the edge of the turbulent core (at a point about two thirds from the center). This is justified in view of the fact that the eddy diffusivity appears to remain fairly constant in the turbulent core. The validity for assumption 3 was partially supported by estimating the convected mass transfer from measurements of the axial concentration gradient and the mean velocity in the region between the wall and the edge of the turbulent core. This estimated convection loss amounted to approximately 5% of the total mass transfer rate at the Reynolds number of 160,000.

The slopes of some of the concentration profiles in the neighborhood of the dry wall appear to be slightly negative, an indication, perhaps, that a transfer of water vapor might be taking place from the "dry" wall into the main air stream. In order to confirm this, an examination was made of the felt surface of the dry wall, which was found to be somewhat

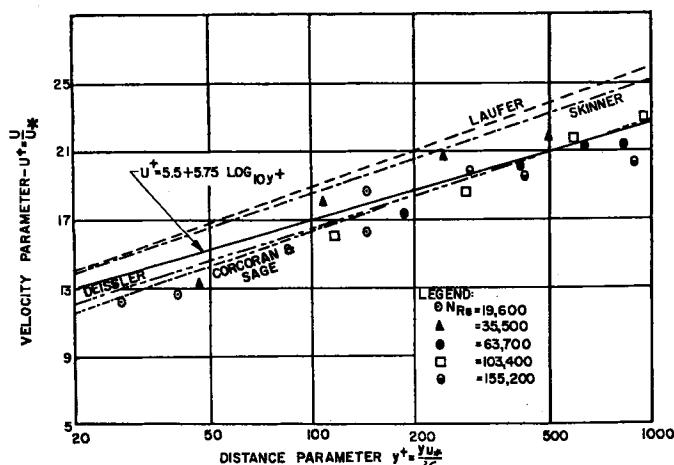


Fig. 13. Comparison with other investigators.

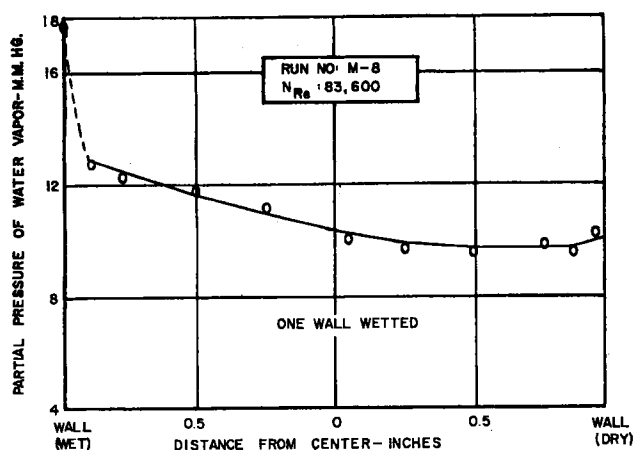


Fig. 14. Typical concentration profile (asymmetric case).

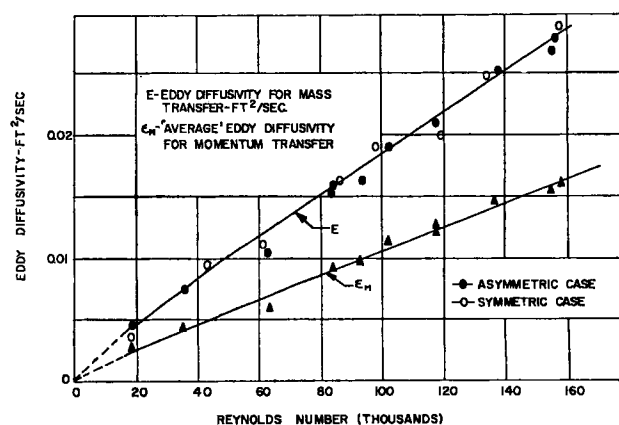


Fig. 15. Eddy diffusivities for mass and momentum transfer.

moist as felt fibers normally tend to absorb moisture.

The correlation of the eddy diffusivity  $E$  with Reynolds number (Figure 15) appears to give an approximately linear relationship. The maximum scatter in the diffusivity values plotted in Figure 15 amounts to about  $\pm 12\%$  from the mean curve, which is within the range of the experimental error.

**Symmetric case.** With both walls wetted, a symmetrical distribution of concentration in the channel cross section results (Figure 16). The eddy-diffusion coefficient is determined in the same manner as outlined for the asymmetric case, except for the fact that in the present case it was obtained from the slopes taken on both sides of the concentration profiles and the individual mass transfer from either wall in the test section. When the two coefficients thus obtained were averaged, the magnitudes of the coefficients from the two sides were found to be approximately equal. The average values obtained in the symmetric case agreed well with those from the asymmetric case. Figure 15

represents the diffusivity values for both cases plotted together.

#### CORRELATION OF DATA

From the fundamental equations of shear stress, the following equation for momentum eddy diffusivity is obtained:

$$\epsilon_M = \frac{g\epsilon}{\rho} = \frac{fU_0^2[1 - (y/b)]}{2(dU/dy)} \quad (1)$$

The velocity gradient ( $dU/dy$ ) may be obtained from the expression for the universal velocity distribution

$$U^+ = 5.5 + 5.75 \log_{10} y^+ \quad (2)$$

The resulting equation for the eddy diffusivity becomes

$$\epsilon_M = 0.4U_0(f/2)^{1/2}y[1 - (y/b)] \quad (3)$$

The experimental data, Figure 17, indicate that the eddy diffusivity remains fairly constant at the maximum value in the main portion of the turbulent core. From the analogy considerations, one gets the following expression for mass

transfer eddy diffusivity from Equation (3):

$$E = 0.0177N_{Re}\nu\sqrt{f} \quad (4)$$

Figure 18 shows the comparison of the experimental data with the values calculated from Equation (4). The friction factor  $f$  is computed from Blasius's equation, the agreement with Equation (4) being excellent at relatively low Reynolds numbers. At high Reynolds numbers the deviation from the predicted curve amounts to a maximum of about  $-12\%$ . In view of the experimental and theoretical assumptions, such a good agreement is fortunate. In Figure 18 the data of Sherwood and Woertz also are plotted for comparison.

From the expression for Reynolds shear stress and the usual definition of the correlation coefficient  $R$ , one obtains

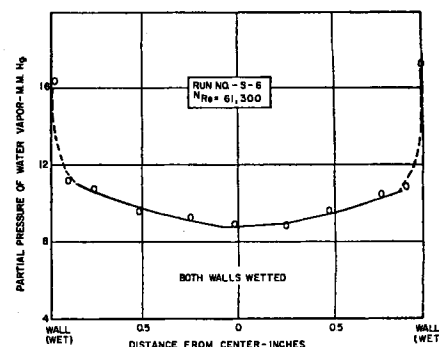


Fig. 16. Typical concentration profile (symmetric case).

$$\epsilon_M = \frac{R(u'^2)^{1/2}(\overline{v'^2})^{1/2}}{dU/dy}$$

Assumptions of isotropic turbulence and of the equality of mass and momentum diffusivities lead to

$$E = \frac{Rv'^2}{dU/dy} \quad (5)$$

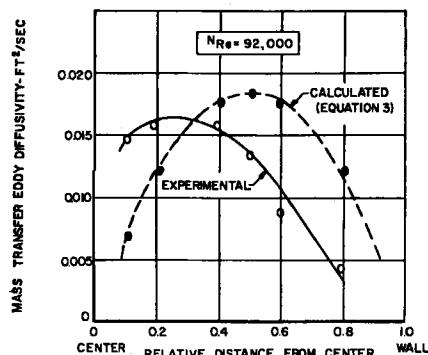


Fig. 17. Typical variation of mass transfer eddy diffusivity across the channel.

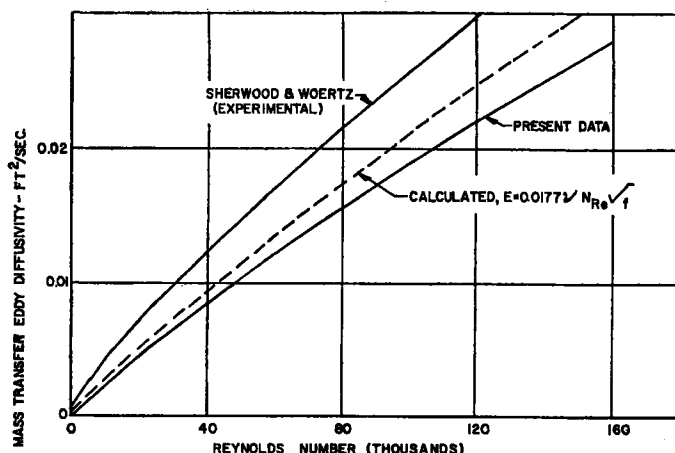


Fig. 18. Comparison of experimental and calculated eddy diffusivity.

where values of  $R$  and  $\bar{v}^2$  are known from the hot-wire data.

Typical variations of these quantities across the channel are represented in Figures 11 and 12. The values of the mass transfer eddy diffusivity are computed from the hot-wire data and the measurement of the velocity distribution. In Figure 17 these values are compared with the variation of  $E$  predicted from Equation (3). Despite the necessary uncertainty in measuring the velocity gradient graphically, the general trend in the two curves appears to be quite similar. The eddy diffusivity remains fairly constant in the main portion of the turbulent core.

## CONCLUSIONS

1. For a fully developed turbulent flow in a wetted-wall channel, mass flux is transferred faster than the flux of momentum (Figure 15).

2. The turbulent exchange coefficients for mass and momentum are approximately linear with Reynolds number in the experimental range of Reynolds numbers from 20,000 to 160,000.

3. The mean value of eddy diffusivity for mass transfer in the turbulent core of a duct can be satisfactorily predicted,

within about 12%, from the correlation equation

$$E = 0.0177 N_{Re} \nu \sqrt{f} \quad (8,000 < N_{Re} < 160,000)$$

## ACKNOWLEDGMENT

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- $t_c$  = temperature at center point of wetted wall, °F.
- $t_w$  = wall temperature, °F.
- $T$  = absolute temperature, °R.
- $u$  = instantaneous velocity in direction  $x$ , ft./sec.
- $u'$  = fluctuating component of velocity  $u$ , ft./sec.
- $U$  = mean velocity in  $x$  direction at any point ( $x, y$ )
- $U_m$  = velocity at center of duct, ft./sec.
- $U_*$  = friction velocity, defined by  $U_* = (\tau_0 g / \rho)^{1/2}$
- $U^+$  = dimensionless velocity, defined by  $U^+ = U / U_*$
- $U_0$  = average velocity in channel, ft./sec.
- $v$  = instantaneous velocity in the direction  $y$ , ft./sec.
- $v'$  = fluctuating component of the velocity  $v$ , ft./sec.
- $y$  = distance from wall of duct, ft.
- $y^+$  = dimensionless distance, defined by  $(y U_*) / \nu$
- $\rho$  = fluid density, lb./cu. ft.
- $\epsilon$  = eddy viscosity, (lb.)/(sec.)/sq. ft.
- $\epsilon_M$  = eddy diffusivity of momentum transfer, sq. ft./sec.
- $\mu$  = molecular viscosity, (lb.)/(sec.)/sq. ft.
- $\tau$  = shear stress on plane parallel to direction of flow, lb./sq. ft.
- $\tau_0$  = shear stress at wall of duct, lb./sq. ft.
- $\nu$  = kinematic viscosity of fluid, sq. ft./sec.

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